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## ABSTRACT

Presented is a method, derived from ordering theory, for the multidimensional scaling of dichotomous item data. The method is said to be related to the methodological multivariate extension of I. Guttman's scalogram analysis developed by C. Coombs and his students. An example is used to compare the data analytic results of the ordering theoretic method and the results of Coombs' method using the conjunctive model. Some relationships of the ordering theoretic method to conventional psychometric data analytic procedures are discussed. (Author/GW)

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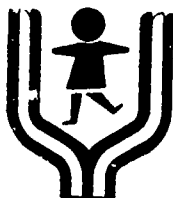
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AN ORDERING-THEORETIC METHOD OF  
MULTIDIMENSIONAL SCALING OF ITEMS

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Research, Development and Demonstration  
Center in Education of Handicapped Children  
Minneapolis, Minnesota

March 1972



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University of Minnesota Research, Development and Demonstration  
Center in Education of Handicapped Children

(Place of publication shown in parentheses where applicable.)

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MULTIDIMENSIONAL SCALING OF ITEMS

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Research, Development and Demonstration  
Center in Education of Handicapped Children  
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RESEARCH AND DEVELOPMENT CENTER  
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The University of Minnesota Research, Development and Demonstration Center in Education of Handicapped Children has been established to concentrate on intervention strategies and materials which develop and improve language and communication skills in young handicapped children.

The long term objective of the Center is to improve the language and communication abilities of handicapped children by means of identification of linguistically and potentially linguistically handicapped children, development and evaluation of intervention strategies with young handicapped children and dissemination of findings and products of benefit to young handicapped children.

An Ordering-Theoretic Method of  
Multidimensional Scaling of Items

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### Abstract

A method for the multidimensional scaling of dichotomous item data is presented which is derived from ordering theory. This method is related to the methodological multivariate extension of Guttman's scalogram analysis developed by Coombs and his students. An example is provided and comparison is provided between the data analytic results of the ordering theoretic method and those of the method of Coombs and his students for their conjunctive model. Some relationships of this method to conventional psychometric data analytic procedures are discussed.

An Ordering-Theoretic Method of  
Multidimensional Scaling of Items

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Ordering theory has been introduced as an alternative model of measurement that makes rich use of boolean algebraic procedures and that serves as an extension of Guttman's scalogram analysis. A qualifying property of data analysis from an ordering-theoretic perspective is that the item response matrix is used not to generate summative scores or square correlation matrices but rather to generate square matrices indicating frequencies of certain item response patterns. For example, Bart and Krus (1973) discussed the use of a square matrix, which indicates the frequencies of (0,1) response patterns for various item pairs, in determining a hierarchy among items. In its present form, ordering theory is restricted to analysis of bivalent items.

In the cited paper by Bart and Krus, it was indicated that the inner-item logical relationship of "is a prerequisite to" can be used to reinterpret the inter-item relationships amidst an ideal Guttman scale. To reveal prerequisite relationships among item pairs, frequencies of disconfirmatory (0,1) response patterns are scrutinized. For example, item A is a prerequisite to item B to the extent that (0,1) response patterns for items A and B respectively has a low frequency of occurrence. For an ideal Guttman scale, item A is a prerequisite to item B if the (0,1) response pattern for items A and B respectively has a zero frequency of occurrence. Items are



ordered in accordance with the prerequisite relation.

As was noted by Torgerson (1958), the primary test for unidimensionality of a given instrument is the transitivity of the ordering relation. In ordering theory the requirement of the transitivity of the ordering relation is satisfied by the prerequisite relation which is transitive. Branches of an ordering of test items, which can with some sets of items be depicted as a hierarchy of items, are chains of linearly ordered items compliant with test characteristics typical for ideal Guttman scales. The type of data analysis that generates an ordering of test items is performed directly on the raw item data and avoids the insertion of summative scores as a phase mediating the raw item response matrix and more advanced treatments of data. The problem of dimensionality and homogeneity of a given set of measures thus can be approached directly by operations carried on the data undistorted by initial summation over such dimensions as those of items and of subjects.

#### Previous multidimensional extensions of Guttman's model

In the area of deterministic models, Clyde Coombs and his students (Coombs and Kao, 1955; Bennett, 1951, 1956; Milholland, 1953) have contributed to an extension of the Guttman model to the multidimensional case. One model in their extension is termed the conjunctive model. In the conjunctive model, a subject passes an item if he is as capable as or more capable than the demands of that item on each and every one of the dimensions of the space for the items. In other words, the subject fails the item if the subject is less

capable than the item difficulty for any one or more dimensions of the item space.

One problem relating to this extension was the determination of the number of dimensions in the space for a set of items. Attempts to estimate the minimum dimensionality of  $n$  stimuli were articulated by Bennett (1951) and Milholland (1953). To exemplify this extension, Torgerson (1958) presented a hypothetical set of data which is supposed to fit the conjunctive model for two dimensions; Table 1 depicts that data in the form of matrix R.

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 Insert Table 1 about here  
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Reconstruction of the order of items in separate dimensions is based on Bennett's theorem (Bennett, 1956). The item arrangement in two dimensions as a result of operations described by Torgerson (1958) is reprinted in Figure 1 and indicates that the item order ECDBA determines one dimension and the item order ABCED determines the other dimension.

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 Insert Figure 1 about here  
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The two-dimensional construction is based on the assumption that data fit into two dimensions as specified by Milholland's formula for a lower bound of item space dimensionality (Milholland, 1953). As can easily be seen, the Coombs' model for this set of data is not completely determined and incompatible patterns such as the item response pattern for subject type 10 are considered to be errant. The determination of the dimensionality of an item space

remains a problem in the extension of the Guttman model.

An ordering-theoretic extension of Guttman's model

To provide comparison with an ordering-theoretic approach to that problem of multidimensional scaling of items, an ordering-theoretic analysis was performed on the data of Table 1. First, items and subject types were rearranged according to decreasing marginal sums. An item pattern matrix A (see Table 2) is then constructed such that cell entry  $a_{ij}$  equals the number of (0,1) response patterns for items i and j respectively; thus, for example, the number 4 entered in the cell for the C row and B column indicates that 4 subject types provided (0,1) response patterns for items C and B respectively. Matrix A is similar to the matrix of percentages of disconfirmatory response patterns for inter-item prerequisite relationships used by Bart and Krus (1973).

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Insert Table 2 about here  
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From Table 1, a subject pattern matrix B is constructed such that cell entry  $b_{ij}$  equals the number of items to each of which subject type i gave a favorable response and subject type j gave an unfavorable response; thus, matrix B which is depicted in Table 3 is analogous to matrix A. To exemplify entries in matrix B, the number 3 entered in the cell for the subject type 2 row and the subject type 7 column indicates that 3 items were answered favorably by subject type 2 while being answered unfavorably by subject type 7.

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 Insert Table 3 about here  
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Using the procedure described by Bart and Krus (ibid.), the hierarchy among the 14 subject types with no tolerance level for disconfirmatory response patterns is depicted in Figure 2.

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 Insert Figure 2 about here  
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From the hierarchy of Figure 2, one could see, for example, that if subject type 7 succeeded on an item in the five-item test, then subject type 6 succeeded on the same item.

The transpose of matrix  $A$ ,  $A^T$ , is then constructed to provide indices of the strengths of various inter-item prerequisite relationships. Matrix  $A^T$  which is shown in Table 4 indicates frequencies of (1, 0) response patterns. Given that for any two items  $i$  and  $j$  there are two primary prerequisite relationships -- namely, the relationship that success on item  $i$  is a prerequisite to success on item  $j$  and the relationship that success on item  $j$  is a prerequisite to success on item  $i$ , the (1,0) response patterns may be viewed as quite informative from an information-theoretic viewpoint for their occurrence tends to reduce the uncertainty regarding the form of prerequisite relationship that may hold between two items (Rao, 1965; Ash, 1965). The (0,0) and (1,1) response patterns do not provide any information as to which of the two primary two-item prerequisite relationships is more tenable for both response patterns confirm

both of the primary prerequisite relationships.

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Insert Table 4 about here

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Matrix  $A^T$  is considered to be analogous to the variance-covariance matrix in factor analysis. The factoring problem then relates to the determination of an independent set of linearly-ordered factors that will be determined by chains of two-item prerequisite relationships.

As a next step, any long branch of the hierarchy for the subject pattern matrix B (Figure 2) can be chosen. For example, start with the branch composed of subject types 5, 6, 7, 8, 9, and 14. Their response patterns are read from the item response matrix of Table 1 and recorded in matrix  $R_1$  of Table 5. Next, their item marginals are computed, items are rearranged in descending order, and a matrix  $A_1^T$  is constructed and cited in Table 5 in a manner similar to the construction of matrix  $A^T$  for the matrix of Table 1. Note that the order of the items in  $A_1^T$  is identical to the DECBA order of items in the second dimension of Torgerson's example of Figure 1. This procedure is repeated for the remaining branches of Figure 2 and is recorded in Table 5 with the designation of matrices  $R_2, A_2^T, R_3, A_3^T, R_4,$  and  $A_4^T$ .

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Insert Table 5 about here

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The row marginals of matrices  $A_1^T$ ,  $A_2^T$ ,  $A_3^T$ , and  $A_4^T$  are then used to form matrix C cited in Table 6 in which cell entry  $c_{ij}$  is the index of the association of item i with factor j. Matrix C is termed an ordering loading matrix. Matrix C can be viewed

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 Insert Table 6 about here  
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as analogous to a factor loading matrix, with column totals analogous to factor contributions and row totals analogous to communalities. Note that the grand sum of 62 equals the number of one-zero changes in columns of matrix R, i.e., in information theory terms, the number of bits accounted for by items. Factor IV is due to the response pattern of one subject type only and can be considered as a residual one. The scale model for the five items in three dimensions as designated by matrix C is presented in Figure 3.

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 Insert Figure 3 about here  
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One can now reconsider the conjunctive model of Figure 1. The ordering theoretic method proposes several improvements over the previous method of data processing in multidimensional extensions of deterministic, Guttman-type models. One improvement entails the exact determination of the number of dimensions based on Thurstone's notion of rank of the matrix as the dimensionality indicator as opposed to the estimation of the lower bound of dimensionality.

Another improvement relates to the presentation of a computational algorithm for a determinate procedure different from the partially indeterminate procedure based on Bennett's theorem.

### Discussion

The ordering-theoretic method of the multidimensional scaling of items that has been discussed is an extension of Guttman's scalogram analysis as are the multidimensional scaling methods of Coombs and others. Also, even though the ordering-theoretic method does not employ correlational procedures as do conventional factor analytic procedures of item data analysis, the ordering-theoretic method has several analogous elements to the factor analytic procedures. The matrix of inter-item (1,0) response pattern frequencies is analogous to a variance-covariance matrix for the same set of items. The ordering loading matrix is analogous to a factor loading matrix. Also, the formulation of this ordering-theoretic method proceeds along lines comparable to those used in the formulation of multiple factor analysis such as those designated by Thurstone (1933). For example, the Thurstone method of using the rank of the correlation matrix to depict the number of factors in the item space has a counterpart in the ordering-theoretic method which is used to determine the number of columns in the ordering loading matrix.

Many issues still remain to be examined with respect to the ordering-theoretic method as ordering theory is still in its early stages of development. For example, some unresolved issues relate

to the determination of analogous methods of factor rotation and the determination of methods to index the degrees of stability of ordering-theoretic factor loadings over time. However, this ordering-theoretic multidimensional scaling method in its present state can be used to scale any set of bivalent test items in a multidimensional manner. Thus, this method is another tool with rich capabilities for the behavioral researcher.



TABLE 1

Hypothetical data matrix for the conjunctive model. (From Torgerson, 1958, p. 350).

		<u>Matrix R</u>					
		Items					
Subject Types		A	B	C	D	E	Row Totals
	1	1	0	0	0	0	1
	2	1	1	0	0	0	2
	3	1	1	0	1	0	3
	4	1	1	1	1	0	4
	5	1	1	1	1	1	5
	6	0	1	1	1	1	4
	7	0	0	1	1	1	3
	8	0	0	0	1	1	2
	9	0	0	0	1	0	1
	10	0	1	0	0	0	1
	11	0	1	0	1	0	2
	12	0	1	1	1	0	3
	13	0	0	1	1	0	2
	14	0	0	0	0	0	0
Column Totals		5	8	6	10	4	33

TABLE 2  
Item pattern matrix A for five items

	D	B	C	A	E	Row Totals
D	0	2	0	2	0	4
B	4	0	2	1	2	9
C	4	4	0	3	1	12
A	7	4	4	0	3	18
E	6	6	3	4	0	19
	21	16	9	10	6	62

TABLE 3

Subject pattern matrix B for 14 subject types

	Subject Types														Row Totals
	5	4	6	3	7	12	2	8	11	13	1	9	10	14	
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	0	1	0	1	0	0	1	0	0	0	0	0	0	4
6	1	1	0	1	0	0	1	0	0	0	1	0	0	0	5
3	2	1	2	0	2	1	0	1	0	1	0	0	0	0	10
7	2	2	1	2	0	1	2	0	1	0	1	0	1	0	13
12	2	1	1	1	1	0	1	1	0	0	1	0	0	0	9
Subject Types 2	3	2	3	1	3	2	0	2	1	2	0	1	0	0	20
8	3	3	2	2	1	2	2	0	1	1	1	0	1	0	19
11	3	2	2	1	2	1	1	1	0	1	1	0	0	0	15
13	3	2	2	2	1	1	2	1	1	0	1	0	1	0	17
1	4	3	4	2	3	3	1	2	2	2	0	1	1	0	28
9	4	3	3	2	2	2	2	1	1	1	1	0	1	0	23
10	4	3	3	2	3	2	1	2	1	2	1	1	0	0	25
14	5	4	4	3	3	3	2	2	2	2	1	1	1	0	33
Column Totals	37	27	28	19	22	18	15	14	10	12	9	4	6	0	221

TABLE 4

Transpose of the item pattern matrix A for five items

Matrix A <sup>T</sup>						Row Totals
	D	B	C	A	E	
D	0	4	4	7	6	21
B	2	0	4	4	6	16
C	0	2	0	4	3	9
A	2	1	3	0	4	10
E	0	2	1	3	0	6
Column Totals	4	9	12	18	19	62

TABLE 5

Item data sub-matrices  $R_1, R_2, R_3, R_4$  and corresponding item pattern matrix transposes  $A_1^T, A_2^T, A_3^T$ , and  $A_4^T$  used in the construction of an ordering loading matrix.

Matrix $R_1$		Matrix $A_1^T$					
		Items					
		D	B	C	A	E	
Subject types	5	1	1	1	1	1	
	6	1	1	1	0	1	
	7	1	0	1	0	1	
	8	1	0	0	0	1	
	9	1	0	0	0	0	
	14	0	0	0	0	0	
Column Totals		5	2	3	1	4	

		Items					Row Totals
		D	E	C	B	A	
Items	D	0	1	2	3	4	10
	E	0	0	1	2	3	6
	C	0	0	0	1	2	3
	B	0	0	0	0	1	1
	A	0	0	0	0	0	0

Matrix $R_2$		Matrix $A_2^T$					
		Items					
		D	B	C	A	E	
Subject types	4	1	1	1	1	0	
	3	1	1	0	1	0	
	2	0	1	0	1	0	
	1	0	0	0	1	0	
Column totals		2	3	1	4	0	

		Items					Row Totals
		A	B	D	C	E	
Items	A	0	1	2	3	4	10
	B	0	0	1	2	3	6
	D	0	0	0	1	2	3
	C	0	0	0	0	1	1
	E	0	0	0	0	0	0

TABLE 5 (cont'd)

		Matrix $R_3$				
		Items				
		D	B	C	A	E
Subject types	12	1	1	1	0	0
	11	1	1	0	0	0
	10	0	1	0	0	0
Column Totals		2	3	1	0	0

		Matrix $A_3^T$					
		Items					
		B	D	C	E	A	Row Totals
Items	B	-	1	2	3	3	9
	D		-	1	2	2	5
	C			-	1	1	2
	E				-	0	0
	A					-	0

		Matrix $R_4$				
		Items				
		D	B	C	A	E
Subject type	13	1	0	1	0	0
Column Totals		1	0	1	0	0

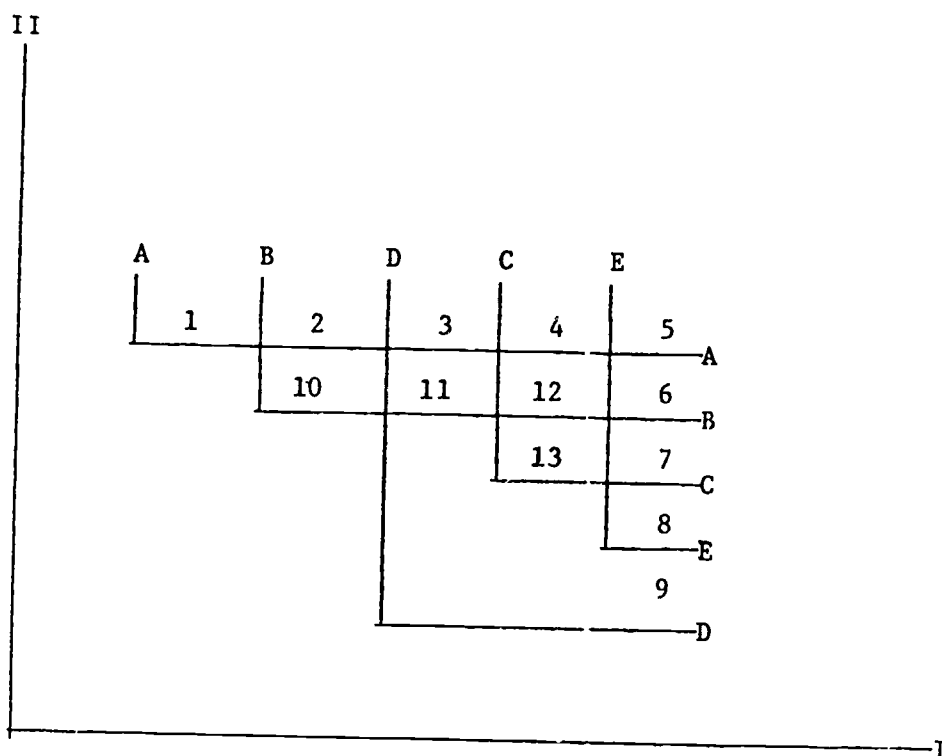
		Matrix $A_4^T$					
		Items					
		D	C	E	B	A	Row Totals
Items	D	-	0	1	1	1	3
	C		-	1	1	1	3
	E			-	0	0	0
	B				-	0	0
	A					-	0

TABLE 6

Ordering loading matrix C for five items

		Factors				Row Totals
		I	II	III	IV	
Items	D	10	3	5	3	21
	E	6	0	0	0	6
	C	3	1	2	3	9
	B	1	6	9	0	16
	A	0	10	0	0	10
Column Totals		20	20	16	6	62

FIGURE 1



Two-dimensional configuration corresponding to the data from Table 1. Dimensions are inferred from Milholland's formula for a lower bound of dimensionality and configuration is reconstructed according to Bennett's theorem. Numbered regions in the figure correspond to response patterns of like numbered subject types in Table 1. (From Torgerson, 1958, p. 350).



FIGURE 2

Ordering theoretic-hierarchy for 14 subject types

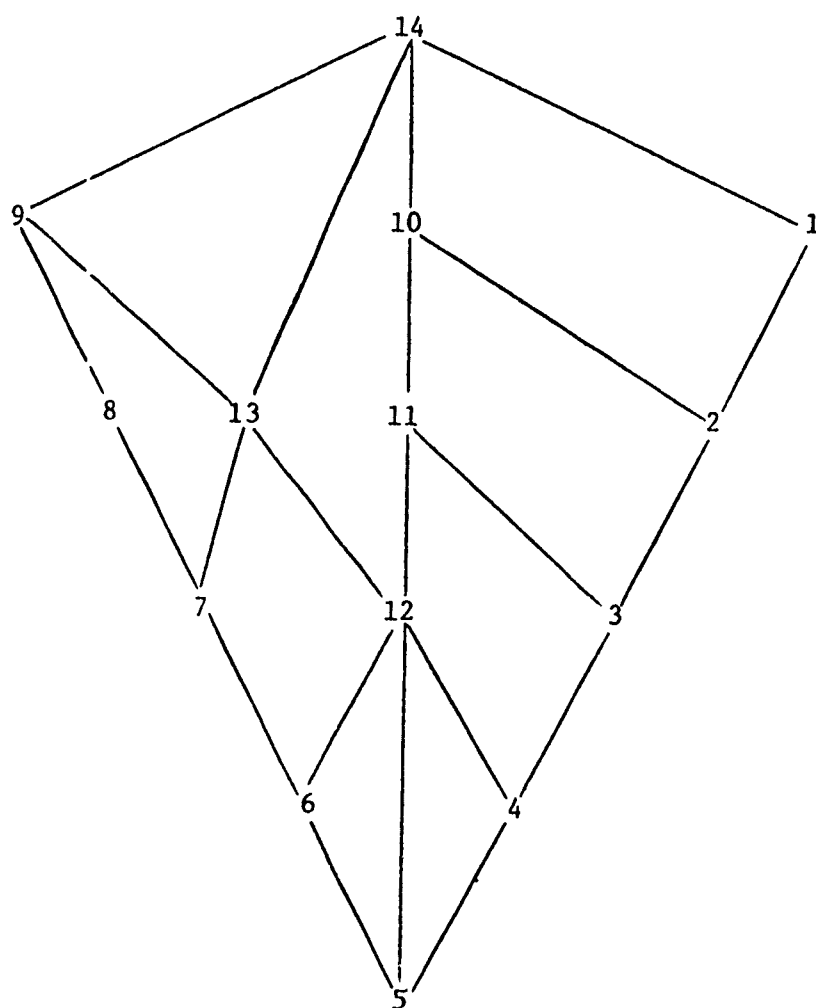
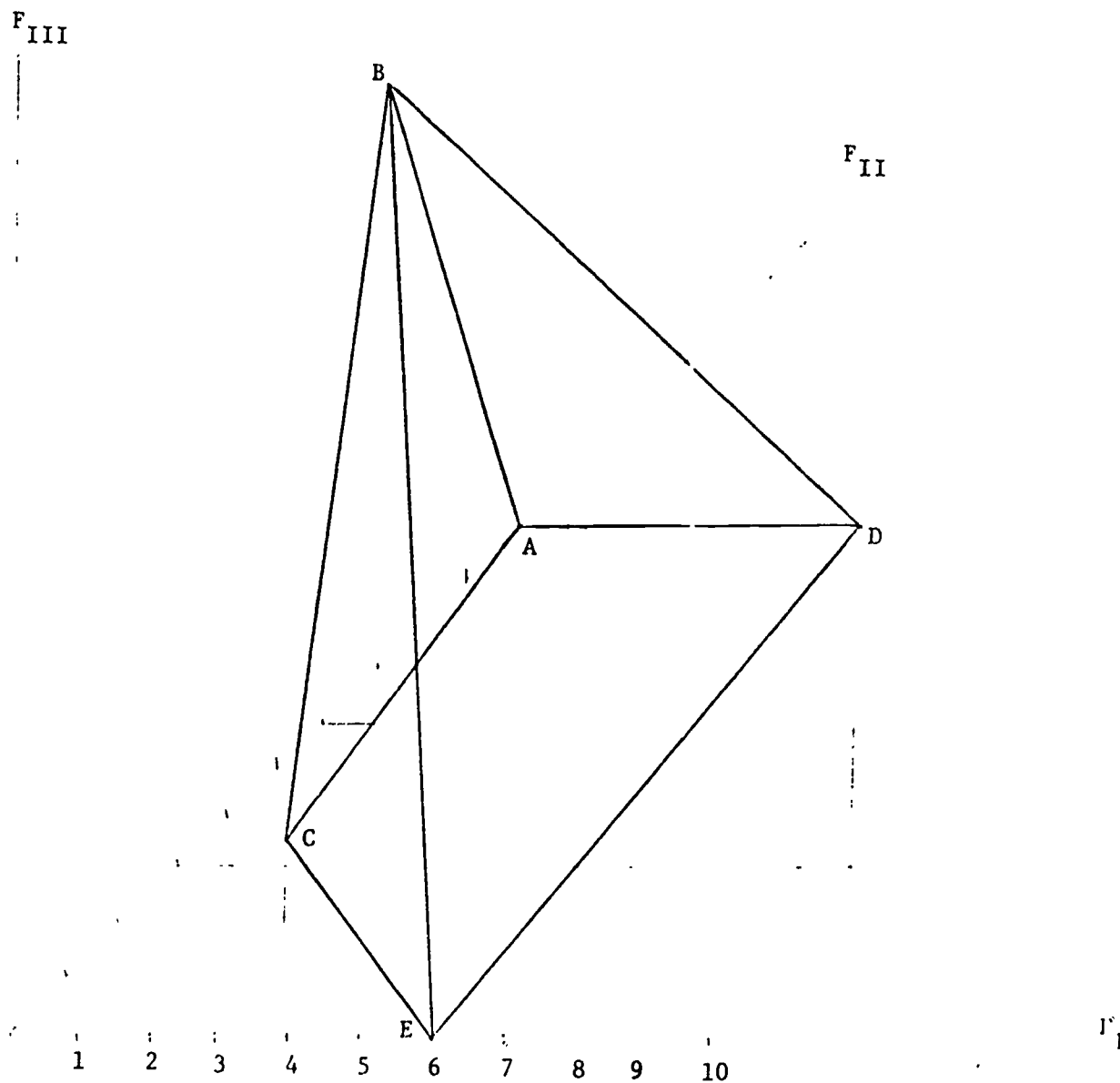


FIGURE 3

Three-dimensional scaling representation for the ordering loading matrix of Table 6.



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#### Footnotes

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